On isomorphism type of some structures in many-one degrees

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Many-one degree (all necessary definitions could be found in [1]) is called simple if it contains simple or computable set. It is also called hypersimple if it contains hypersimple or computable set. We say that m-degree is $\Delta^0_2$-degree if it consists of the sets from class $\Delta^0_2$ of arithmetical hierarchy.

It is easy to show that hypersimple m-degrees form an ideal in simple m-degrees; simple m-degrees form an ideal in c.e. m-degrees; c.e. m-degrees form a principal ideal in $\Delta^0_2$ m-degrees and $\Delta^0_2$ m-degrees form an ideal in the upper semilattice of all m-degrees. So the sets of hypersimple, simple, computably enumerable and $\Delta^0_2$ m-degrees are all the distributive upper semilattices w.r.t. m-reducibility relation. Straightforward construction shows that the upper semilattice of $\Delta^0_2$ m-degrees has no greatest element.

Now it has been proved [3] that the next upper semilattices are isomorphic:

1. the upper semilattice of all hypersimple m-degrees;
2. the upper semilattice of all simple m-degrees;
3. the upper semilattice of all c.e m-degrees with the greatest element excluded;
4. the upper semilattice of all $\Delta^0_2$ m-degrees;
5. Rogers semilattices of $\Sigma^0_2$-computable numberings of finite nontrivial family of the $\Sigma^0_2$-sets which are pairwise incomparable w.r.t. inclusion.

The existence of isomorphism between semilattices in items 3 and 4 was firstly announced by S. D. Denisov in 1978 [2] but the proof of this fact has never been published.

References

3. Podzorov, S.Yu.: Universal Lachlan’s semilattice with the greatest element excluded. Algebra and Logic, to be printed.